

**Original scientific paper**

*Received: 11.4.2019*

*Accepted: 10.12.2019*

**UDK: 674.07:684.4]:[004.942:536.21**

**COMPUTATION OF THERMAL CONDUCTIVITY OF FLAT WOOD DETAILS  
IN A MODEL OF THEIR ONE SIDED HEATING BEFORE BENDING**

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**ABSTRACT**

Mathematical descriptions of thermal conductivity  $\lambda_w$ , specific heat capacity  $c_w$ , heat transfer coefficient  $\alpha_w$ , and density  $\rho_w$  of non-frozen wood in hygroscopic range have been introduced in own 1D non-linear mathematical model of one sided conductive heating process of flat wood details. For the numerical solution of the model a software program has been prepared in the calculation environment of Visual FORTRAN Professional. By means of the program, the 1D non-stationary temperature distribution along the thickness of subjected to one sided conductive heating flat wood details, aimed at their plasticizing in the production of curved back parts of chairs, has been calculated. The change of  $\lambda_w$  for beech details with an initial temperature of 20 °C, moisture content of 0.15 kg<sup>-1</sup>, and thicknesses of 12 mm, 16 mm, and 20 mm during their 30-min. one sided heating at temperature of 100 °C of the heating metal body has also been computed, visualized and analyzed.

**ey words:** flat wood details, one sided heating, thermal conductivity, plasticizing, bending

**1. INTRODUCTION**

An important component of the technologies for production of curved wood details is their plasticizing up to the stage that allows their faultless bending (Kavalov and Rusanov 2000, Taylor 2001, Trebula and Klement 2002, Videlov 2003, Pervan 2009, Angelski 2010, Deliiski and Dzurenda 2010, Gaff and Prokein 2011).

One sided heating is used for plasticizing wood in the production of curved details for the back parts of chairs, or outwards curved parts for the corpses of string music instruments (violins, violoncellos, guitars, mandolins). The technologies for plasticizing such details are most often carried out in specific equipment used for bending.

The curved details for the back parts of chairs, which are produced by this method of plasticizing, have a relatively small thickness  $h_w$ , a large radius  $R$  of the curvature, and a ratio of  $R/h_w = 20 \div 25$  (Kavalov and Angelski 2014).

For such heating of details with thicknesses between 10 and 25 mm and moisture content from 12% to 20% in the process of production of chairs, hot hydraulic presses in the range from 80 °C to 120 °C with appropriately bent surfaces or electrically heated metal body are usually used.

The duration of the heating process and the energy consumption for one sided heating of wood details aimed at their plasticizing before bending, depends on many factors: wood species, thickness and moisture content of the details, temperatures of the heating body and of the surrounding air, desired degree of plasticizing, etc. (Rice and Lucas 2003, Gaff and Prokein 2011).

It is a common knowledge that wood thermal conductivity  $\lambda_w$  characterizes the intensity of the heat distribution in wood materials. Because of this, for calculation of the one sided heating process of wood details at given initial and boundary conditions, it is a must to know  $\lambda_w$  of non-frozen wood.

The aim of this work is to study the changes in thermal conductivities of subjected to one sided conductive heating flat beech details aimed at their plasticizing in the production of curved back parts of chairs. The study has to be performed by means of own 1D non-linear model of the non-stationary temperature distribution along the thickness of the details during their one sided heating, in which a mathematical description of the wood thermal conductivity depending on the temperature, wood density and moisture content, is introduced.

## 2. MATERIAL AND METODS

### 2.1. Mechanism of Temperature Distribution in Flat Wood Details Subjected to One Sided Heating

The mechanism of heat distribution in wood details during their one sided conductive heating can be described by the equation of the heat conduction. When the width and length of the wood details exceed their thickness by at least 3 and 5 times respectively, then calculation of change in the temperature only along the thickness of the details in the centre of their flat side during the one sided heating (i.e. along the coordinate  $x$ , which coincides with the details' thickness  $h_w$ ) can be carried out applying the following non-linear 1D mathematical model (Deliiski et al. 2018):

$$c_w(T, u) \rho_w(\rho_b, u, u_{fsp}, S_v) \frac{\partial T_w(x, \tau)}{\partial \tau} = \lambda_w(T, u, \rho_b) \frac{\partial^2 T_w(x, \tau)}{\partial x^2} + \frac{\partial \lambda_w(T, u, \rho_b)}{\partial T} \left( \frac{\partial T_w}{\partial x} \right)^2 \quad (1)$$

with an initial condition

$$T_w(x, 0) = T_{w0} \quad (2)$$

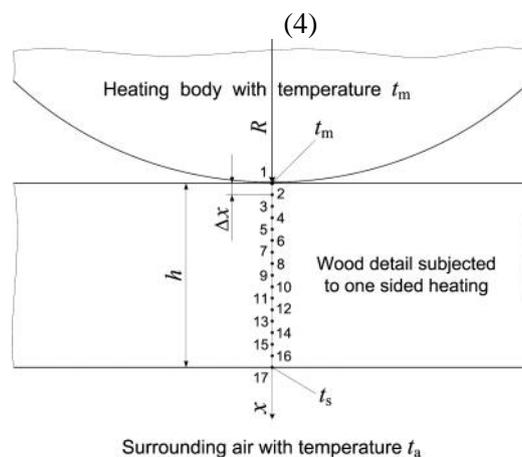
and following boundary conditions:

- from the side of the details' heating – at prescribed surface temperature, which is equal to the temperature of the metal heating body  $t_m$  (see Fig. 1 below):

$$T_w(0, \tau) = t_m(\tau), \quad (3)$$

- from the opposite non-heated side of the details – at convective heat exchange between the details' surface and the surrounding air environment

$$\frac{dT_w(x, \tau)}{dx} = - \frac{\alpha_w(\tau)}{\lambda_{ws}(\tau)} [T_a(\tau) - T_{ws}(\tau)]. \quad (4)$$



**Figure 1.** Positioning of the nodes of the calculation mesh for solution of the model (1) ÷ (4) along the thickness of the wood detail subjected to one side heating

According to eq. (3), the temperature at the details surface being in contact with the heating body (i.e. the point with *-coordinate = 0 m*) is equal to its temperature  $T_m$  due to the extremely high coefficient of heat transfer between the body and the wood during their very close contact.

Pursuant to eq. (4), the temperature of the non-heated surface of the details depends on the current value of the difference between this temperature and the temperature of the surrounding air, as well as on the current values of the thermo-physical characteristics of the wood  $\rho_w$  and  $c_w$  during the one sided heating.

## **2.2. Mathematical Description of the Thermo-Physical Characteristics of the Wood during One Sided Heating of the Details**

For the usage of the equations (1) and (4) it is needed to have a mathematical description of the specific heat capacity of the wood,  $c_w$ , of the wood's thermal conductivity,  $\lambda_w$ , and of the wood density,  $\rho_w$ , and also of the heat transfer coefficient of the non-heated side of the details,  $\alpha_w$ . The following equations for the calculation of  $c_w$ ,  $\lambda_w$ , and  $\rho_w$  of non-frozen wood in the hygroscopic range (i.e. when  $u < u_{fsp}$ ) have been suggested in (Deliiski 2003, 2011, 2013):

$$c_w = \frac{2097u + 826}{1 + u} + \frac{9.92u + 2.55}{1 + u}T + \frac{0.0002}{1 + u}T^2, \quad (5)$$

$$\lambda_w = \lambda_{w0}[1 + \beta(T - 273.15)], \quad (6)$$

where

$$\lambda_{w0} = K_{ad} \cdot v \cdot [0.165 + (1.39 + 3.8u) \cdot (3.3 \cdot 10^{-7} \rho_b^2 + 1.015 \cdot 10^{-3} \rho_b)], \quad (7)$$

$$v = 0.15 - 0.07u, \quad (8)$$

$$\beta = (2.05 + 4u) \cdot \left( \frac{579}{\rho_b} - 0.124 \right) \cdot 10^{-3}, \quad (9)$$

$$\rho_w = \rho_b \frac{1 + u}{1 - \frac{S_v}{100} (u_{fsp}^{293.15} - u)}. \quad (10)$$

Equations (5) ÷ (10) are part of the mathematical description of the thermo-physical characteristics of wood, which has been done earlier (Deliiski 2003) using many experimental data derived by different scientists.

It is pointed out and shown (Deliiski 2011, 2013) that this data is widely used in both European and American specialized literature when calculating various processes of thermal treatment of wood. This fact ensures the appropriateness of the non-linear mathematical model suggested in this paper to the real process of warming up flat wood details during their one sided heating before bending.

Calculation of the heat transfer coefficient  $\alpha_w$  can be conducted using the following equation, which is valid for the case of cooling of horizontally situated wood plates in atmospheric conditions of free convection in the hygroscopic range (Deliiski et al. 2018):

$$\alpha_w = 3.256 [ \rho_w(\tau) - T_a(\tau) ]^{0.25}. \quad (11)$$

### 3. RESULTS AND DISCUSSION

For numerical solution of the discrete analogue of the mathematical model (1) ÷ (10), a software program has been prepared in FORTRAN, which was input in the calculation environment of Visual Fortran Professional developed by Microsoft.

Using the program, computations have been made so as to determine the 1D change of the temperature in flat beech details with thicknesses  $h_w = 0.012$  m,  $h_w = 0.016$  m,  $h_w = 0.020$  m, initial temperature  $t_{w0} = 20$  °C, and moisture contents  $u = 0.15$  kg·kg<sup>-1</sup>, during their 30-min. one sided heating with  $t_m = 100$  °C and at  $t_a = 20$  °C. The following values of the parameters of the beech details subjected to heating were used during the computations: basic density  $\rho_b = 560$  kg·m<sup>-3</sup>, coefficient in eq. (7) cross sectional to the fibers  $K_{ad-cr} = 1.28$ , volume shrinkage  $S_v = 17.3$  %, and fiber saturation point  $u_{fsp}^{293.15} = 0.31$  kg·kg<sup>-1</sup> (Videlov 2003, Deliiski and Dzurenda 2010).

By means of the software program, computations have also been carried out for determination of change of the following variables during the details' heating:

- average mass temperature of the details according to the equation

$$t_{w-avg} = \frac{1}{h_w} \int_{(h)} T_w(x, \tau) dx, \tag{12}$$

- average mass thermal conductivity of the details according to the modified equation (6)

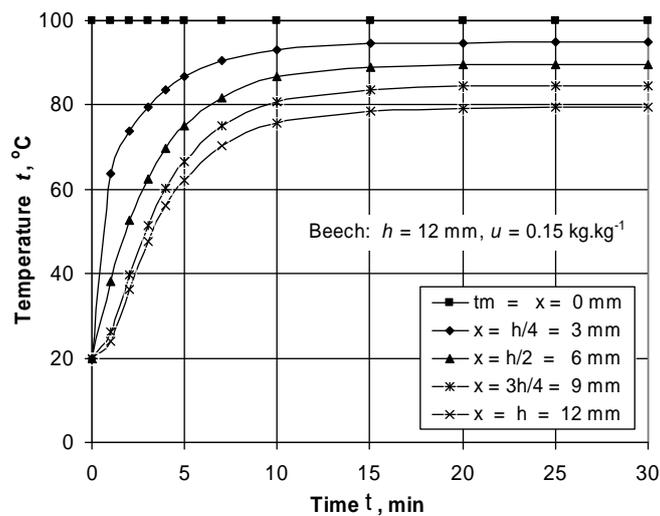
$$\lambda_{w-avg} = \lambda_{w0} [1 + \beta(T_{w-avg} - 273.15)], \tag{13}$$

- thermal conductivity of the non-heated surface of the details according to the equation

$$\lambda_{ws} = \lambda_{w0} [1 + \beta(T_{ws} - 273.15)]. \tag{14}$$

Figure 2 presents the change of temperature  $t_w$ , calculated by the model in 4 equidistant from other points along the thicknesses of the beech detail with  $h_w = 12$  mm during its one sided heating at  $t_m = 100$  °C. The coordinates of those points are shown in the legend of the figure.

Figure 3 presents the calculated change of the average mass temperature  $t_{w-avg}$  during the one sided heating at  $t_m = 100$  °C of the studied beech details, depending on  $h_w$ .



**Figure 2.** Change in  $t_w$  along the thickness of beech detail with  $h_w = 12$  mm during its one sided heating at  $t_m = 100$  °C

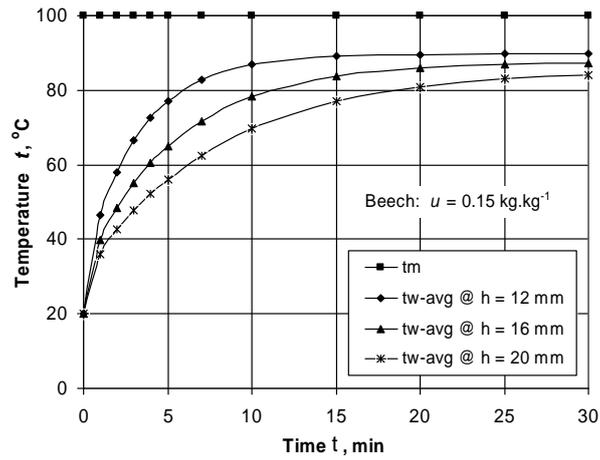


Figure 3. Change in  $t_{w-avg}$  of beech details during their one sided heating at  $t_m = 100\text{ }^\circ\text{C}$ , depending on  $h_w$

Figure 4 and Figure 5 present the calculated change of thermal conductivities  $\lambda_{w-avg}$  and  $\lambda_{ws}$  respectively during the one sided heating at  $t_m = 100\text{ }^\circ\text{C}$  of the studied beech details, depending on  $h_w$ .

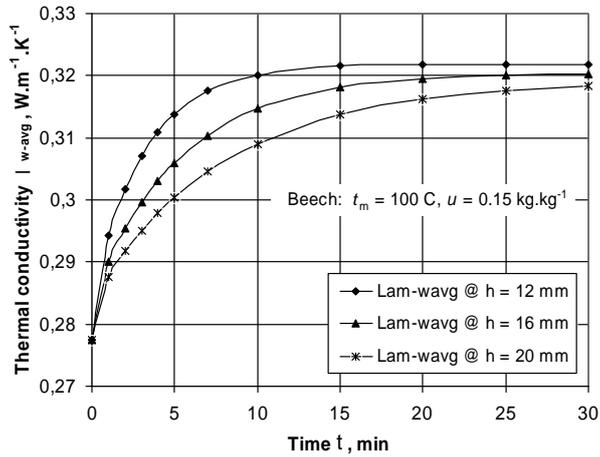


Figure 4. Change in  $\lambda_{w-avg}$  of beech details during their one sided heating at  $t_m = 100\text{ }^\circ\text{C}$ , depending on  $h_w$

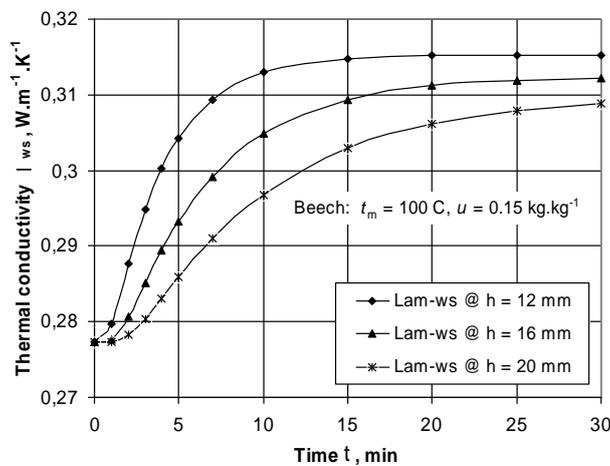


Figure 5. Change in  $\lambda_{ws}$  of beech details during their one sided heating at  $t_m = 100\text{ }^\circ\text{C}$ , depending on  $h_w$

The results obtained show that during the unilateral heating of the details, there is change in all studied parameters of the process according to complex curves.

By increasing the heating time, the curves of  $t_w$  gradually approach asymptotically their highest values, decreasingly dependent on the remoteness of the characteristic points from the heated surface of the details. Analogously, the curves of the change in  $t_{w-avg}$ ,  $\lambda_{w-avg}$ , and  $\lambda_{ws}$  approach asymptotically their highest values, decreasingly dependent on  $h_w$ .

The largest values of  $t_w$ ,  $t_{w-avg}$ ,  $\lambda_{w-avg}$ , and  $\lambda_{ws}$  are achieved when a stationary temperature distribution occurs along the details' thickness.

At  $t_m = 100^\circ$  the most slowly changing temperature of the detail's surface,  $t_{ws}$ , that is in contact with the outside air environment with  $t_a = 20^\circ\text{C}$  reaches temperatures of  $50^\circ$ ,  $60^\circ$ , and  $70^\circ$  which are necessary for commencement of bending of the studied details with different radii  $R$  (Kavalov and Angelski 2014) after a duration of the one sided heating at  $h_w = 12$  mm, equal to 3.3 min. (for  $50^\circ\text{C}$ ), 4.6 min (for  $60^\circ\text{C}$ ), and 6.9 min. (for  $70^\circ\text{C}$ ) (see Fig. 2)

The decreasingly dependent on  $h_w$  values of  $t_{w-avg}$  reach at the end of 30 min heating the following values:  $89.7^\circ\text{C}$  at  $h_w = 12$  mm,  $87.2^\circ\text{C}$  at  $h_w = 16$  mm, and  $84.2^\circ\text{C}$  at  $h_w = 20$  mm.

At the beginning of the heating process the beech details have an initial temperature of  $20^\circ\text{C}$ . At this temperature the thermal conductivities  $\lambda_{w-avg}$  and  $\lambda_{ws}$  of all details are equal to  $0.2774 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$ .

At the end of the 30-min one sided heating of the details,  $\lambda_{w-avg}$  and  $\lambda_{ws}$  reach the following values:

- for details with  $h_w = 12$  mm:  $\lambda_{w-avg} = 0.3218 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$  at  $t_{w-avg} = 89.7^\circ\text{C}$  and  $\lambda_{ws} = 0.3153 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$  at  $t_{ws} = 79.4^\circ\text{C}$ ;
- for details with  $h_w = 16$  mm:  $\lambda_{w-avg} = 0.3202 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$  at  $t_{w-avg} = 87.2^\circ\text{C}$  and  $\lambda_{ws} = 0.3122 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$  at  $t_{ws} = 74.6^\circ\text{C}$ ;
- for details with  $h_w = 20$  mm:  $\lambda_{w-avg} = 0.3183 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$  at  $t_{w-avg} = 84.2^\circ\text{C}$  and  $\lambda_{ws} = 0.3089 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$  at  $t_{ws} = 69.4^\circ\text{C}$ .

#### 4. CONCLUSIONS

This paper shows and analyzes diagrams of 1D non-stationary change of temperature along the thickness of flat beech details subjected to unilateral heating in order to be plasticized before their bending for production of curved back parts of chairs. It presents also diagrams of the change in the average mass temperature  $t_{w-avg}$ , average thermal conductivity of the details  $\lambda_{w-avg}$ , and thermal conductivity of the non-heated surface of the details  $\lambda_{ws}$  during their one sided heating.

The diagrams are drawn up in compliance with the results calculated by means of non-linear mathematical model, which has been presented in the present paper. For numerical solution of the model, a software program was prepared in FORTRAN in the calculation environment of Visual Fortran Professional developed by Microsoft.

The 1D distribution of temperature and the change of  $t_{w-avg}$ ,  $\lambda_{w-avg}$ , and  $\lambda_{ws}$  for flat beech details with thickness of 12 mm, 16 mm, and 20 mm, at initial wood temperature of  $20^\circ\text{C}$ , and moisture content of  $0.15 \text{ kg}\cdot\text{kg}^{-1}$  during their one sided heating for a period of 30 min at a temperature of the heating metal body  $t_m = 100^\circ\text{C}$  and temperature of the air near the non-heated side of the details  $t_a = 20^\circ\text{C}$  is calculated, visualized and analyzed using the model.

The results obtained from computer solutions of the model could be used for the following purposes:

- Visualization and technological analysis of the temperature change along the thickness of details from different wood species with different thickness and moisture content during their one sided heating before bending accomplished by a heating body with different temperatures;
- To determine the duration of details heating, which is necessary for achieving the minimum required plasticity of the details before their bending with a specified radius;
- Creation of a scientifically derived model-based automatic control of the one sided heating process (Hadjiski and Deliiski 2016, Hadjiski et al. 2018).

## **Symbols**

- $c$  = specific heat capacity ( $\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$ )  
 $h$  = thickness (m)  
 $R$  = radius of bending of the heated and plasticized wood details (m)  
 $S$  = wood shrinkage (%)  
 $t$  = temperature ( $^{\circ}\text{C}$ ):  $t = T - 273.15$   
 $T$  = temperature (K):  $T = t + 273.15$   
 $u$  = moisture content ( $\text{kg}\cdot\text{kg}^{-1}$ )  
 $x$  = coordinate along the thickness of the details subjected to heating:  $0 \leq x \leq X = h_w$  (m)  
= heat transfer coefficient ( $\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ )  
= thermal conductivity ( $\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$ )  
= density ( $\text{kg}\cdot\text{m}^{-3}$ )  
= time (s)  
 $x$  = step on the  $x$ -coordinate of the model, which coincides with the thickness of the wood details (m) subjected to heating

## **Subscripts and Superscripts**

- a = air  
ad = anatomical direction of the wood  
avg = average (for mass temperature of the wood details at given moment of their heating)  
b = basic (for wood density, based on dry mass divided to green volume)  
cr = cross-sectional to the fibers  
fsp = fiber saturation point  
m = medium (for the temperature of the heating metal body used for unilateral heating)  
p = process (for the duration of whole process of the one sided heating)  
s = surface (for non-heated surface of the subjected to heating wood details)  
v = volume (for the wood shrinkage)  
w = wood  
0 = initial (for average mass temperature of the details in the beginning of their heating or for time level in the beginning of the calculations)  
293.15 = at 293.15 K, i.e. at  $20^{\circ}\text{C}$  (for the standardized values of the wood fiber saturation point)

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