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## INVESTIGATION OF THE NATURAL FREQUENCIES AND THE MODE SHAPES OF CIRCULAR SAW WITH COMPENSATING SLOTS AND LOW NOISE SLOTS BY THE FINITE ELEMENT METHOD

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#### ABSTRACT

This paper shows the methods and results of the simulative investigations of circular saw with compensating and low noise slots. The investigations are an extension of the previous ones of the authors. The natural frequencies and mode shapes of this kind of circular saws are obtained as results of the investigations. The estimation was done by the application programme Cosmos Works. The physical and mechanical properties of the materials were taken into account. The adequate mechanic-mathematical model was used for the aims of the study. The typical characteristics of the structure of this kind of circular saws were taken into account in the model. The circular saw was drawn in 3D by the application programme Solid Works and it was modelled with four nodes 3D finite elements. The results of this investigation prove the practical significance of the model. They point to the possibilities of determining resonant regimes and the results are a basis for their detailed studying.

Key words: circular saws, modelling, vibrations

## **1. INTRODUCTION**

Some high-level noise and vibrations appear during the work of circular machines. The task of meeting the high requirements concerning the level of the noise and vibrations in the woodworking and furniture production demands introduction of adequate standards for limiting noise levels and increasing these standards. Therefore, it is necessary to investigate the reasons for vibrations in the machines and to study the dynamical forces and dynamical processes in them. The usage of current materials and technologies is a premise for designing a circular saws which has the qualities that are necessary for its operation. These saws allow intensification of the work process, using higher cutting speed. This process has advantages but there are some problems. One of them is the high level of heating of the saw during its operation. It can lead to deformations of the disk and damage the accuracy and quality of the processing of the product. In order to avoid that, it is necessary to introduce compensating slots in the saw's body.

Another important problem is the high level of vibrations and the noise. Therefore, low noise slots are made in order to limit the above mentioned disadvantages. A circular saw with compensating and low noise slots is shown in Fig. 1. The excellent mechanic characteristic of high-quality steel, which is used in production of circular saws, allows foreseeing the necessary slots that improve the work in the saw during their constructive formation.

The existence of these slots in the circular saw's body influences the frequencies of the natural vibrations and mode shapes of the circular saw. This impacts the tasks of designing and measuring of the whole circular machine (Vukov, Gochev, Slavov, 2010); (Vukov and Georgieva, 2009); (Obreshkov, 1996).

It is necessary to do in advance some estimations related to the danger of occurrence of resonance.

Therefore, it's necessary to investigate the natural frequencies and mode shapes of the circular saw. It leads to simulative investigations which help to differentiate between the resonant regimes (Amirouche, 2006); (Coutinho, 2001). They are done on basis of the adequate mechanic-mathematical model, taking into account the typical characteristics of the structure and the physical – mechanical characteristics of the materials of this kind of circular saws (Veits, Kochura, Martinenko, 1971); (Minchev and Grigorov, 1998); (Filipov, 1977).

The aim of the study is to build an adequate mechanicalmathematical model for investigation of free vibrations of a kind of circular saws with compensating and low noise slots, concerning the characteristics in its structure. Some simulative investigations can be made on this basis and these investigations can help to define resonance regimes and to formulate some requirements needed for aboiding them.



*Figure 1.* Circular saw with compensating and low noise slots

(1)

# 2. MECHANIC-MATHEMATICAL MODEL OF CIRCULAR SAW WITH COMPENSATING AND LOW NOISE SLOTS

A circular saw with compensating and low noise slots, drawn in 3D by the application programme Solid Works (<u>www.solidworks.com</u>), is shown on the Fig. 2.

Fig. 3 shows the mesh of four node 3D finite elements, modelled by the application programme Cosmos Works.



Figure 2. A circular saw with compensating and low noise slots, drawn in 3D

The investigation of the vibrations of the circular saw with compensating and low noise slots requires formulation and solution of the differential equations which describe these processes. Therefore, matrix mechanics was applied (Angelov, 2010); (Angelov and Slavov, 2010).

#### **DIFFERENTIAL EQUATIONS**

The differential equations which describe the free continuous vibrations of the circular saw are as follows:

$$\mathbf{M}.\ddot{\mathbf{q}} + \mathbf{C}.\mathbf{q} = \mathbf{0},$$

Where:

$$\mathbf{q} = \begin{bmatrix} q_1 & q_2 & \dots & q_n \end{bmatrix}^T$$
 is the vector of the generalized coordinates; (2)

 $\mathbf{M}$  – the matrix, which characterizes the mass-inertial properties of the mechanical system;

 $\boldsymbol{C}$  – the matrix, which characterizes the elastic properties of the mechanical system.

The system of connected linear differential equations is obtained when the vibrations are small. Particular solutions of the system of the differential equations (1) are searched as:

$$\mathbf{q}_r = h_r . \sin(\omega_r . t + \varphi),$$

(3)

Where  $h_r$  the amplitude of the small vibration on the generalized is coordinate  $q_r$  with natural frequency  $\omega_r$ , and  $\varphi$  is initial phase.



Figure 3. The circular saw with compensating and low noise slots, modelled by the mesh of finite elements

After differentiation of (3) and substituting in (1), a system of linear algebraic equations is obtained:

$$\left| \mathbf{C} - \boldsymbol{\omega}^2 \cdot \mathbf{M} \right| \cdot \mathbf{V} = 0 \,. \tag{4}$$

To determine the natural frequencies and the mode shapes, it is necessary to solve the task of finding the natural values and the natural vectors of the equations (4). Doing the equations (4) requires the following:

$$\det\left(\mathbf{C}-\boldsymbol{\omega}^{2}.\mathbf{M}\right)=0.$$
(5)

The roots of the characteristics equation determine the natural frequencies. The natural frequencies form the matrix of the natural values. It is:

$$\omega = diag\left[\omega_{r,r}\right], \quad r = 1, 2, \dots n.$$
(6)

The natural frequencies are determinate by (6):

$$f_r = \frac{\omega_{r,r}}{2.\pi} \quad Hz \,. \tag{7}$$

The natural values of the system (5) determine the natural vectors. The modal matrix of the free vibrations is determined by the equations (4) and (5)

$$\mathbf{V} = \begin{bmatrix} V_{11} & V_{12} & \dots & V_{1n} \\ V_{21} & V_{22} & \dots & V_{2n} \\ \dots & \dots & \dots & \dots \\ V_{m1} & V_{m2} & \dots & V_{mn} \end{bmatrix} \qquad i = 1...m; \quad j = 1..n,$$
(8)

 $V_{ij}$  are the unknown amplitudes of the nodes' moving by free vibrations. The natural frequencies and the mode shapes are determined by the known matrix, which characterizes the mass-inertial properties and the matrix that characterizes the elastic properties of the mechanical system.

#### NUMERICAL INVESTIGATIONS

The numerical investigations are done by modelling of a kind of circular saws with compensating and low noise slots by the finite elements method. Physical- mechanical characteristics of materials are taken into account – they are shown in Tables 1, 2 and 3. The estimation of the natural frequencies and mode shapes of the circular saw are done by the application programme Cosmos Works.

Document Name and Reference	Treated As	Volumetric Properties
Extrude 1	Solid Body	Mass:1.08623 kg Volume:0.000138197 m^3 Density:7860 kg/m^3 Weight:10.6451 N

Table 1. Model Information

Table 2. Mesh Information

Mesh type	Solid Mesh
Mesher Used:	Standard mesh
Jacobian points	4 Points
Mesh Quality	High

Table 3. Mesh Information - Details

Total Nodes	432920
Total Elements	246068
Maximum Aspect Ratio	62.116
% of elements with Aspect Ratio < 3	98.6
% of elements with Aspect Ratio > 10	0.026
% of distorted elements(Jacobian)	0
Time to complete mesh (hh;mm;ss):	00:04:10

## **3. RESULTS**

The first 30 natural frequencies and mode shapes of the studied circular saw were determined. The estimated natural frequencies are shown in the Table 4. The results, which illustrate only some of the natural modes, are not included because of the limited size of the article. They are shown in Figure 4 and their characteristics and shown in the Table 5.

Frequency Number	Rad/sec	Hertz	Seconds
1	0	0	1e+032
2	0	0	1e+032
3	0	0	1e+032
4	0	0	1e+032
5	0	0	921.85
6	0	0	482.23
7	756.19	120.35	0.008309
8	779.85	124.12	0.0080569
9	1240	197.35	0.0050671
10	1704.7	271.31	0.0036858
11	1706.5	271.6	0.0036818
12	2816.5	448.25	0.0022309
13	2819.4	448.73	0.0022285
14	2839.8	451.97	0.0022125
15	2949.8	469.47	0.00213
16	4083.2	649.86	0.0015388
17	4253.2	676.92	0.0014773
18	4278	680.87	0.0014687
19	5187.6	825.63	0.0012112
20	5264.6	837.88	0.0011935
21	5736.3	912.97	0.0010953
22	5921.2	942.39	0.0010611
23	6192.6	985.58	0.0010146
24	6224.6	990.67	0.0010094
25	6849.9	1090.2	0.00091726
26	7222.7	1149.5	0.00086992
27	7311.3	1163.6	0.00085937
28	7347.9	1169.4	0.00085511
29	7576.9	1205.9	0.00082926
30	7627.1	1213.9	0.0008238

Table 4. Mode List



Figure 4. Mode Shapes (a)



Figure 4. Mode Shapes (b)

Name	Туре	Min	Max
Displacement 1	URES: Resultant Displacement Plot for Mode Shape: 1 (Value = 0 Hz)	959.484 mm, Node: 1	959.484 mm, Node: 1
Displacement 7	Mode Shape: 7	0.00672513 mm,	2256.27 mm,
	(Value = 120.351 Hz)	Node: 283546	Node: 1447
Displacement 8	Mode Shape: 8	0.00487282 mm,	2260.07 mm,
	(Value = 124.117 Hz)	Node: 311117	Node: 1527
Displacement 9	Mode Shape: 9	0.00354176 mm,	1958.89 mm,
	(Value = 197.353 Hz)	Node: 374402	Node: 151812
Displacement 10	Mode Shape: 10	0.0647264 mm,	2638.08 mm,
	(Value = 271.311 Hz)	Node: 374688	Node: 1362
Displacement 11	Mode Shape: 11	0.024532 mm,	2597.49 mm,
	(Value = 271.604 Hz)	Node: 399359	Node: 306
Displacement 12	Mode Shape: 12	0.0309914 mm,	3017.9 mm,
	(Value = 448.253 Hz)	Node: 331861	Node: 153389
Displacement 14	Mode Shape: 14	0.00786593 mm,	2729.96 mm,
	(Value = 451.971Hz)	Node: 374104	Node: 962
Displacement 15	Mode Shape: 15	0.00438254 mm,	2922.58 mm,
	(Value = 469.473 Hz)	Node: 383031	Node: 111
Displacement 16	Mode Shape: 16	0.00560711 mm,	5151.75 mm,
	(Value = 649.863 Hz)	Node: 285930	Node: 3743
Displacement 17	Mode Shape: 17	0.00250359 mm,	3196.22 mm,
	(Value = 676.922 Hz)	Node: 54840	Node: 1317
Displacement 19	Mode Shape: 19	0.0702759 mm,	2396.44 mm,
	(Value = 825.634 Hz)	Node: 421334	Node: 636
Displacement 20	Mode Shape: 20	0.0118645 mm,	5621.35 mm,
	(Value = 837.882 Hz)	Node: 301168	Node: 153392
Displacement 21	Mode Shape: 21	0.0234711 mm,	3677.91 mm,
	(Value = 912.966 Hz)	Node: 362254	Node: 116
Displacement 22	Mode Shape: 22	0.00476746 mm,	4235.89 mm,
	(Value = 942.39Hz)	Node: 415799	Node: 451
Displacement 23	Mode Shape: 23	0.0060148 mm,	12439.1 mm,
	(Value = 985.58 Hz)	Node: 386607	Node: 3743
Displacement 26	Mode Shape: 26	0.00747722 mm,	7652.83 mm,
	(Value = 1149.53 Hz)	Node: 336046	Node: 6974
Displacement 28	Mode Shape: 28	0.0148316 mm,	16072.6 mm,
	(Value = 1169.45 Hz)	Node: 257740	Node: 7610
Displacement 29	Mode Shape: 29	0.0147381 mm,	10227.2 mm,
	(Value = 1205.89 Hz)	Node: 318578	Node: 6434
Displacement 30	Mode Shape: 30	0.0136241 mm,	8770.68 mm,
	(Value = 1213.89 Hz)	Node: 378962	Node: 4802

Table 5. Characteristics of the natural modes

#### **4. CONCLUSIONS**

The paper presents the method and results of simulative investigations of the circular saw with compensating and low noise slots. The natural frequencies and mode shapes of the studied circular saw are obtained. The estimation is done by a current application programme, taking into account the typical characteristics in the construction of a kind of circular saws and the physical - mechanical characteristics of their materials. The results of the investigations prove the practical significance of the developed mechanic-mathematical model and the methods for study of the circular saw with compensating and low noise slots. They point to the possibilities of determining the resonant regimes and they are a basis for their detail studying.

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