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ANALYSES AND VALIDATION OF CUTTING FORCES PREDICTION MODELS IN WOOD MACHINING

Marija Djurković, Gradimir Danon

University of Belgrade, Faculty of Forestry, Department of Wood Science and Technology,
11030 Belgrade, Serbia
**Corresponding author e-mail: marija.djurkovic@sfb.bg.ac.rs*

ABSTRACT

A lot of research has been done related to the influence of different machining factors on the wood machining process. Methodologically, the factors are divided into three main groups that affect cutting mechanics: factors associated with material properties, factors that are dependent on the cutting tools, and factors attributed to the cutting process itself. A better understanding of machining factors could be the foundation for predicting the behavior of the material in the machining process. Also, it may consequently contribute to a more efficient and economical machining outcome and above all, the better quality of the machined surface. One of the basic parameters of cutting mechanics is cutting forces. There are various models in literature for determining the dependence of values of cutting forces on the selected impact factors. Different cutting force models are analyzed and compared in this paper. The results performed in the peripheral milling parallel with wood grains of oak wood (*Quercus robur*) were used for testing the models. The analysis of these models indicated that there is no match between the calculated and the experimental results, but there is a similarity in the form of the curve. Changes in the measured values are accompanied by corresponding changes in the calculated values, which indicates that these models can find application in real cutting conditions.

Keywords: machining factors, peripheral milling, cutting force, cutting power, oak wood.

1. INTRODUCTION

Cutting process is influenced by a number of factors which could be divided into three main groups that affect cutting mechanics: factors associated with material, factors that are dependent on the cutting tools, and factors attributed to the cutting process itself. Factors which are dependent on material include wood species, physical and mechanical properties, moisture content and wood temperature. Factors which are associated with cutting tool include cutting angle, edge round up radius, physical and mechanical properties of tool material and roughness of the cutting edge. Finally, the third group of impact factors connected with cutting process itself includes chip thickness, wood cutting mode (longitudinal, transverse, tangential, or combined), cutting and clearance angle, cutting speed, friction between tools and work-piece and tool vibrations.

Estimation of the optimum values of the factors that influence cutting mechanics could be the foundation for predicting the behavior of the material in the machining process. Also, it may consequently contribute to more efficient and economical machining outcomes considering the fact that energy and raw material costs are rising. And above all, it may affect positively the quality of the machined surface, since the improvement in surface quality is becoming increasingly important (Krenke, Frybort, Müller 2017a, b).

One of the basic parameters of cutting mechanics is cutting forces. Better understanding of the machining factors' influence on cutting forces could ensure optimization of the machining process. Proper optimization of the cutting process requires an appropriate approach to assessing the cutting forces, as this could help for better understanding of the interaction between tools and raw materials (Orlowski *et al.* 2020).

Generally, in real cutting conditions, high values of cutting forces are associated with tool vibrations and defects of the part. Therefore, modeling and simulation of cutting forces before the very cutting process could be a strong support to manufacturers to set the cutting regime to stay within the machine spindle power range, verify their fixture, and optimize both the material and cycle time (related to the energy consumed during the process).

There are many models in literature determining the dependence of values of cutting forces on the selected impact factors. One of the simplest methods for predicting cutting forces is the method of coefficients (Orlicz 1982, Zubčević 1988, Goglia 1994, Kršljak 2013). In this type of model the authors start from the referent unit cutting resistance for a particular wood species measured under accurately defined and controlled conditions. What makes the model simple is the fact that the calculations of the cutting force require data on the measured unit cutting resistance, cutting conditions, easily available data on the material, tools, and respective tables. This type of model has certain weaknesses, which could be seen in differences obtained comparing measured and calculated cutting forces values (Mandic *et al.* 2014; Djurkovic and Danon 2017). This phenomenon can be explained by the fact that physical and mechanical properties of wood are insufficiently and inadequately included in the models. In order to improve the model, other properties should be included such as anatomical, physical and mechanical ones: mode of wood cutting (longitudinal, transverse, tangential, or combined), wood density and moisture content, bending strength, tensile strength, hardness etc. (Eyma 2004).

The comparison of measured and calculated values of cutting forces has been analyzed in the study of Djurkovic and Danon (2017). The authors calculated cutting forces based on the values of the required measured cutting power, and then compared them with the cutting forces values obtained applying the method of coefficients (Kršljak 2013) and Axelsson model (Axelsson, Lundberg, Grönlund, 1993). The Axelsson model presents a more accurate modeling of the cutting process. The empirical equation of Axelsson model involves more factors such as: wood density, wood moisture content, wood temperature, cutting speed, mean chip thickness, the angle between the cutting speed vector and the wood grain orientation. The analysis indicated that there is no match between the results, but there is a similarity in the form of a curve, i.e. changes in the measured values are accompanied by corresponding changes in the calculated values. This means that the analyzed models are not suitable for quantification of the cutting forces, but can be used to compare different cutting modes.

More complex models for predicting cutting forces are dedicated to a single machining process (such as circular sawing or milling), including the above mentioned properties (Naylor*et et al.* 2012, Porankiewicz *et al.* 2011, Mandić, Porankiewicz, Danon 2015, Porankiewicz *et al.* 2021, Curti *et al.* 2021). The Naylor model (Naylor *et al.* 2012), included wood density and moisture content, milling depth and wood mechanical properties: bending strength and shear strength, modulus of elasticity and modulus of shear, and toughness. The disadvantages of this model are the cutting conditions that differed greatly compared to those in practice (cutting speed was not higher than at 0.1 $m \cdot s^{-1}$). The Porankiewicz model (Porankiewicz *et al.* 2011) involves physical and mechanical properties of wood, tool characteristics and cutting mode parameters. The study provides statistical equations for tangential and normal, cutting forces in the function of the angle between cutting direction and grain orientation, radius of cutter blade roundness, the values of rake angle, mean chip thicknesses, cutting speeds, wood moisture content, wood density at 8% moisture content, and temperature of wood. The Mandić model (Mandić, Porankiewicz, Danon 2015) presents non-linear, multi variable dependency between the main (tangential) force, and the machining parameters and properties of Pedunculate oak. This model includes density, moisture content, Brinell hardness, bending strength, modulus of elasticity, feed rate per tooth, rake angle, and cutting depth. Porankiewicz *et al.* (2021) included elasticity modules in their predictive model for the cutting force. The study indicated that the tangential force is affected by cutting depth, feed rate per tooth, rake angle, and elasticity modulus which described mechanical properties of wood very well (elasticity modulus by stretching along grains, elasticity modulus by stretching perpendicular to grains, modulus of elasticity by compression along grains, and modulus of elasticity by compression perpendicular to grains). The equations provided in the models gave a strong link between the observed and predicted cutting forces and can be used to analyze the impact of specific inputs on the predicted cutting forces. Both models (Mandić, Porankiewicz, Danon 2015, Porankiewicz *et al.* 2021) were performed during opened, up-milling and peripheral milling process. The other parameters, in addition to the above, including cutting edge round up radius, cutting speed, diameter of the cutter, cutter width, and the number of cutting edges, were kept constant.

2. EXPERIMENT

The experimental setting is described and explained in details in the work of Djurkovic and Danon (2017). Experimental results of cutting forces for comparison with the calculated values according to the models were obtained by testing on oak wood samples (1000mm x 30 mm x 200 mm) at the Center for Wood Processing Machines and Tools, Faculty of Forestry, Belgrade. Before the experiment all samples were conditioned at the temperature of 20±20 °C and relative air humidity of 65±5%.

The experiment was performed using a table-mounted milling cutter MiniMax CU410K (Italy) equipped with a Maggi Engineering feeding device Vario Feed (Italy) with a range of speeds 3-24 m/min. Testing was performed during the peripheral, open, up-milling process. The tools used were three milling cutters manufactured by Freud (Italy). The cutters were equipped with four blades soldered plates, made of hard metal cemented carbide with diameter D = 125 mm, width B = 40 mm. Testing was done with a constant number of rotations per minute of the working spindle (RPM= 5.860), i.e., at a constant cutting speed of 38.35 $m\cdot s^{-1}$. The values of the other processing parameters are shown in Table 1.

Table 1. *The Peripheral Milling Process Parameters of Oak*

Feed speed v_F ($m\cdot min^{-1}$)	Feed per tooth f_Z (mm)	Cutting depth c_D (mm)	Rake angle γ_F(°)	Cutting angle δ(°)
4	0.171	2	16	74
8	0.341	3	20	70
16	0.683	4.5	25	65
* The cutters had clearance angle of α_F= 15°				

Cutting powers required for milling were measured using acquisition device SRD1, indirectly by measuring the power input of the machine driving the electric motor. The acquisition device was equipped with the Power Expert software for analysis, processing, and storage of the results (Mandić and Danon 2010).

3. RESULTS AND DISCUSSION

As explained in detail in the study of Djurkovic and Danon (2017), mean values of measured cutting powers on the oak samples' (with mean measured oven dry densities 725 kg/m^3 and mean measured moisture content 7.28%) were used to calculate the mean values of the main cutting resistance by applying the appropriate formula as follows:

$$F_{sr} = \frac{P_{sr}}{v_r} \qquad 1)$$

where F_{sr} is mean value of the main cutting resistance for one cutter revolution, P_{sr} is mean cutting power [W], v_r is cutting speed [m/s].

Calculated value of the mean force F_{sr}, represents the mean force for one cutter revolution, which means that it also includes idle feed between the blades (Figure 1).

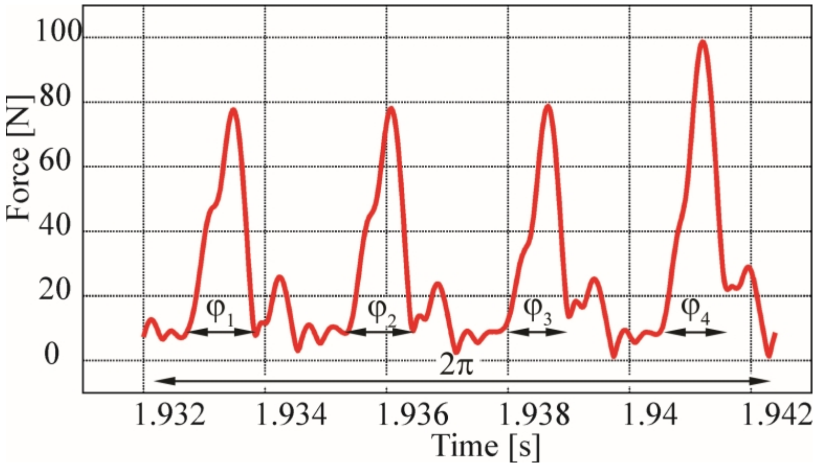

Figure 1. *Change of the main cutting force for one cutter revolution (Djurkovic and Danon 2017)*

In order to obtain mean specific force for cutter F_b engagement, the previous formula needs to be corrected, and it is done as follows:

$$F_b = \frac{F_t \cdot O_{gl}}{\sum_1^4 l_{r1}} = \frac{F_t \cdot 2 \cdot \text{®} \cdot R}{4 \cdot \text{¨}_0 \cdot R} = \frac{F_t \cdot \text{®}}{2 \cdot \text{¨}_0} \tag{2}$$

where O_{gl} - circumference of the milling cutter [m], l_{rl} – engagement length of a single blade and work piece [m], N – number of milling cutter blades (N=4), R – milling cutter radius [m], ϕ_0 – mean angle of blade engagement and work piece.

The values of measured cutting forces are given below (Table 2).

Table 2. *Measured values of wood cutting forces*

NO	$D_{8\%}$	γ	u	C_D	f_Z	e_m	φ_s	$F_{measured}$
	g/cm^3	°	%	mm	mm	mm	rad	N
1	0.785	25	7.25	4.5	0.171	0.032	1.379	42.155
2	0.776	16	7.06	2	0.683	0.086	1.439	99.041
3	0.738	16	6.79	4.5	0.171	0.032	1.379	45.606
4	0.724	20	7.31	3	0.341	0.053	1.413	61.392
5	0.744	25	7.41	2	0.683	0.086	1.439	65.246
6	0.778	16	6.86	2	0.171	0.022	1.443	36.288
7	0.753	20	7.51	3	0.341	0.053	1.413	61.509
8	0.718	25	6.67	4.5	0.683	0.130	1.375	88.131
9	0.735	20	7.14	3	0.341	0.053	1.413	58.159
10	0.744	20	7.31	3	0.341	0.053	1.413	64.453
11	0.692	16	7.70	2	0.683	0.086	1.439	75.082
12	0.741	16	7.83	4.5	0.171	0.032	1.379	43.916
13	0.737	16	6.86	4.5	0.683	0.130	1.375	113.288
14	0.774	20	7.39	3	0.341	0.053	1.413	63.923
15	0.777	20	6.94	3	0.341	0.053	1.413	60.185
16	0.783	25	7.48	4.5	0.171	0.032	1.379	41.975
17	0.755	20	7.25	3	0.341	0.053	1.413	59.966

NO	$D_{8\%}$	γ	u	C_D	f_Z	e_m	φ_s	$F_{measured}$
	g/cm^3	°	%	mm	mm	mm	rad	N
18	0.753	25	7.46	2	0.683	0.086	1.439	67.478
19	0.734	25	7.31	4.5	0.683	0.130	1.375	89.444
20	0.773	16	7.55	2	0.171	0.032	1.379	46.363

The calculated mean forces fall within 36.288 – 113.288 N range, depending on the cutting conditions and physical properties of the wood being cut.

3.1. Cutting force calculation by Kršljak model - the method of coefficients

The specific resistance for specific material and specific cutting conditions (*K*) is obtained when referent unit cutting resistances (K_{el}) are multiplied by corresponding correction coefficients (C_i) calculated in advance, and that can be found in the respective tables (Kršljak, 2013). The formula for calculating wood specific resistance, according to Kršljak, has the following form:

$$K = K_{el} \cdot C_{vr} \cdot C_u \cdot C_{¢} \cdot C_{..} \cdot C_e \cdot C_v \cdot C_{\circ} \quad 3)$$

where K – wood specific resistance for specific cutting conditions, K_{el} – wood specific resistance for e = 1 mm (e is mean chip thickness), C_{vr} – correction factor for wood species C_u– correction factor for wood moisture content, C_e– correction factor for chip thickness, C_δ– correction factor for the cutting angle, C_ϕ – correction factor for penetration angle into the wood, C_v– correction factor for cutting speed, C_ρ – correction factor for the bluntness of a cutting edge.

The magnitude of the main cutting resistance is obtained when the calculated coefficient *K* is multiplied by the cross-sectional surface of a separate particle/chip for a corresponding type of cutting A_s:

$$F = K \cdot A_s \quad (4)$$

Tabular values of correction coefficients and values of calculated specific resistances and cutting forces are given below (Table 3).

Table 3. *Calculated wood specific resistance and cutting forces- model Kršljak*

NO	K_{e1}	C_{vr}	$\mathbf{C_u}$	$\mathbf{C_{esr}}$	$\mathbf{C_\delta}$	$\mathbf{C_\varphi}$	$\mathbf{C_v}$	$\mathbf{C_\circ}$	K	$F_{krsljak}$
	N/mm^2	-	-	-	-	-	-	-	N/mm^2	N
1	14	1.55	1.10	3.08	1.5	0.83	1.28	1.00	117.16	113.88
2	14	1.55	1.10	2.22	2	0.79	1.28	1.00	107.17	277.46
3	14	1.55	1.10	3.08	2	0.83	1.28	1.00	156.21	151.84
4	14	1.55	1.10	2.62	1.7	0.81	1.28	1.00	110.23	174.93
5	14	1.55	1.10	2.22	1.5	0.79	1.28	1.00	80.38	208.10
6	14	1.55	1.10	3.53	2	0.79	1.28	1.00	170.41	110.43
7	14	1.55	1.10	2.62	1.7	0.81	1.28	1.00	110.23	174.93
8	14	1.55	1.10	1.94	1.5	0.83	1.28	1.00	73.80	286.70
9	14	1.55	1.10	2.62	1.7	0.81	1.28	1.00	110.23	174.93
10	14	1.55	1.10	2.62	1.7	0.81	1.28	1.00	110.23	174.93
11	14	1.55	1.10	2.22	2	0.79	1.28	1.00	107.17	277.46
12	14	1.55	1.10	3.08	2	0.83	1.28	1.00	156.21	151.84
13	14	1.55	1.10	1.94	2	0.83	1.28	1.00	98.39	382.26
14	14	1.55	1.10	2.62	1.7	0.81	1.28	1.00	110.23	174.93
15	14	1.55	1.10	2.62	1.7	0.81	1.28	1.00	110.23	174.93
16	14	1.55	1.10	3.08	1.5	0.83	1.28	1.00	117.16	113.88
17	14	1.55	1.10	2.62	1.7	0.81	1.28	1.00	110.23	174.93

NO	K_{e1}	C_{vr}	C_u	C_{esr}	C_δ	C_φ	C_v	$C_\circ$	K	$F_{krsljak}$
	N/mm^2	-	-	-	-	-	-	-	N/mm^2	N
18	14	1.55	1.10	2.22	1.5	0.79	1.28	1.00	80.38	208.10
19	14	1.55	1.10	1.94	1.5	0.83	1.28	1.00	73.80	286.70
20	14	1.55	1.10	3.53	2	0.79	1.28	1.00	170.41	110.43

It is noticeable that specific resistance is most strongly affected by the change in mean chip thickness and blade penetration angle into the wood, while other factors are constant. For the examined cutting conditions the minimum value of calculated cutting force was 110.43 N and the maximum value was 382.26 N according to this model.

3.2. Cutting force calculation by Axelsson model

The equation for Axelsson model (Axelsson, Lundberg, Grönlund, 1993), based on a multifactorial experiment of woodcutting with a circular saw, has the following form:

$$F_p = \left[\frac{-7.37 + e_m \times (0.38 \cdot D_8 - 224.5 \cdot \gamma) + 15.61 \cdot \varphi_s - 2.6 \cdot \varphi_s^3 + 1.31 \cdot \rho + 0.2 \cdot v_r + u \cdot (0.3 \cdot \varphi_s - 0.01 \cdot t)}{4.25} \right] \qquad (5)$$

where: Fp - specific main cutting force ($N \cdot mm^{-1}$), e_m– mean chip thickness (mm), ρ_8 – wood density at 8% moisture content ($kg \cdot m^{-3}$), γ –rake angle (rad), φ_s – the angle between cutting direction and wood grain orientation (work), ρ – radius of cutter blade roundness (μm), u– moisture content (%), v_r – cutting speed ($m \cdot s^{-1}$), t – temperature (°C).

The model includes influences such as material properties, cutting conditions, sharpening angles and blade condition, environmental conditions, which must contribute to achieving more realistic results.Table 4 contains calculated cutting forces and appropriate input data for the model such as: mean chip thickness, wood density at 8% moisture content, rake angle, cutting speed, wood moisture content, environment temperature and radius of the cutter tip roundness.

Table 4. *Calculated wood cutting forces –Axelsson model*

NO	e_m	$D_{8\%}$	γ	φ_s	ρ	u	v	t	$F_{axelsson}^*$
	mm	g/cm^3	rad	rad	μm	%	m/s	^oC	N
1	0.032	0.785	0.4361	1.379	2	7.25	38.35	20	181.17
2	0.086	0.776	0.2791	1.439	2	7.06	38.35	20	277.61
3	0.032	0.738	0.2791	1.379	2	6.79	38.35	20	184.41
4	0.053	0.724	0.3489	1.413	2	7.31	38.35	20	209.57
5	0.086	0.744	0.4361	1.439	2	7.41	38.35	20	249.18
6	0.022	0.778	0.2791	1.443	2	6.86	38.35	20	171.22
7	0.053	0.753	0.3489	1.413	2	7.51	38.35	20	213.98
8	0.130	0.718	0.4361	1.375	2	6.67	38.35	20	294.37
9	0.053	0.735	0.3489	1.413	2	7.14	38.35	20	210.88
10	0.053	0.744	0.3489	1.413	2	7.31	38.35	20	212.42
11	0.086	0.692	0.2791	1.439	2	7.70	38.35	20	259.10
12	0.032	0.741	0.2791	1.379	2	7.83	38.35	20	186.28

NO	e_m	$D_{8\%}$	γ	φ_s	ρ	u	v	t	$F_{axelsson}^*$
	mm	g/cm³	rad	rad	µm	%	m/s	°C	N
13	0.130	0.737	0.2791	1.375	2	6.86	38.35	20	333.49
14	0.053	0.774	0.3489	1.413	2	7.39	38.35	20	216.75
15	0.053	0.777	0.3489	1.413	2	6.94	38.35	20	216.46
16	0.032	0.783	0.4361	1.379	2	7.48	38.35	20	181.38
17	0.053	0.755	0.3489	1.413	2	7.25	38.35	20	213.81
18	0.086	0.753	0.4361	1.439	2	7.46	38.35	20	251.52
19	0.130	0.734	0.4361	1.375	2	7.31	38.35	20	300.96
20	0.032	0.773	0.2791	1.379	2	7.55	38.35	20	188.63

It is obvious that some of the input data have a constant value for all observed cases such as cutting speed, rounded blades and environment temperature at the time of tests. The other values changed depending on the physical properties of the samples and cutting regime, which resulted in a fairly wide range of calculated values for cutting forces (the minimum value of calculated cutting force was 171.22 N and the maximum value was 333.49 N according to this model).

3.3. Cutting force calculation by Mandić model

The Mandić model (Mandić, Porankiewicz, Danon 2015) includes more input data for prediction of the cutting forces, compared with the previous two. The authors give multivariable non-linear dependency between the main (tangential) force, F_C, and the machining parameters and properties of oak (*Quercus robur*) during straight edge, peripheral milling. Tangential force, F_C, was found to be influenced by density D, moisture content m_C, Brinell hardness, H, bending strength, R_B, the modulus of elasticity, E, feed rate per tooth, f_Z, rake angle, γ_F, and cutting depth, C_D:

$$F_C^P = A+B+C \tag{6}$$

where:

$$A = 0.01174 \cdot D^{0.68806} \cdot m_C^{\;0.62019} \cdot H^{\,0.18212} \cdot R_B^{\;0.045741} \cdot E^{0.27236} \cdot f_Z^{\;0.53737} \yen_F^{\;0.54972} \cdot c_D^{\;-0.50384}$$

$$B = 106.55693 \cdot f_Z \cdot c_D - 14.2737 \cdot f_Z \yen_F - 1.80803 \cdot R_b \cdot f_Z - 7.0966 \cdot 10^{-3} \cdot R_B \yen_F$$

$$C = -7.53491 \cdot 10^{-6} \cdot D \cdot E - 0.056067 \cdot D \cdot f_Z + 0.038487 \cdot D_8 \cdot f + 0.01174 \cdot m_C \cdot R_B - 14.60027$$

Table 5 contains calculated cutting forces and appropriate input data for the Mandić model.

***Table 5.** Calculated wood cutting forces – Mandić model*

NO	$D_{8\%}$	γ_F	u	C_D	f_Z	H	R_B	E	$F_{mandić}$
	g/cm³	°	%	mm	mm	N/mm²	N/mm²	N/mm²	N
1	0.785	25	7.25	4.5	0.171	47.78	123.44	12408.65	18.64
2	0.776	16	7.06	2	0.683	47.67	105.27	10924.48	87.41
3	0.738	16	6.79	4.5	0.171	45.79	134.23	11575.01	13.07
4	0.724	20	7.31	3	0.341	42.18	112.32	10806.37	51.01
5	0.744	25	7.41	2	0.683	41.99	120.36	11386.25	60.86
6	0.778	16	6.86	2	0.171	46.81	122.97	13455.73	18.32
7	0.753	20	7.51	3	0.341	37.83	126.44	11267.36	44.75
8	0.718	25	6.67	4.5	0.683	37.57	114.26	10326.45	86.94

NO	$D_{8\%}$	γ_F	u	C_D	f_Z	H	R_B	E	$F_{mandić}$
	g/cm^3	°	%	mm	mm	N/mm^2	N/mm^2	N/mm^2	N
9	0.735	20	7.14	3	0.341	37.11	110.25	11486.40	45.29
10	0.744	20	7.31	3	0.341	45.21	126.81	10773.06	46.89
11	0.692	16	7.70	2	0.683	38.24	108.75	9455.03	68.34
12	0.741	16	7.83	4.5	0.171	40.20	116.84	12118.24	24.09
13	0.737	16	6.86	4.5	0.683	36.85	139.10	12234.84	103.93
14	0.774	20	7.39	3	0.341	44.44	124.28	11304.22	51.74
15	0.777	20	6.94	3	0.341	46.42	116.11	11062.85	50.10
16	0.783	25	7.48	4.5	0.171	43.42	128.20	11001.15	21.46
17	0.755	20	7.25	3	0.341	41.54	117.80	11220.91	43.47
18	0.753	25	7.46	2	0.683	44.30	136.64	12828.28	55.65
19	0.734	25	7.31	4.5	0.683	42.79	136.74	11782.58	83.85
20	0.773	16	7.55	2	0.171	46.39	130.43	10660.37	32.11

For the examined cutting conditions the minimum value of calculated cutting force was 13.07 N and the maximum value was 103.93 N according to this model.

3.4. Cutting force calculation by Porankiewicz model

Porankiewicz model (Porankiewicz *et al.* 2021) represents upgrade of Mandić model since the research was conducted with the same samples and it was extended with the elasticity modulus. This model gives multivariable dependence between the main (tangential) cutting force, F_C, processing parameters and modules of elasticity of oak wood during peripheral milling with a straight edge. The analysis indicated that the tangential force, F_C, is affected by cutting depth, c_D, feed rate per tooth, f_Z, rake angle, γ_F, elasticity modulus by stretching along grains, E_{SA}, elasticity modulus by stretching perpendicular to grains, E_{SP}, modulus of elasticity by compression along grains, E_{CA}, and the modulus of elasticity by compression perpendicular to grains, E_{CP}, as presented in the following formula:

$$F_C{}^P = 0.005392 \cdot e^A + B + C \tag{7}$$

where:

$$A = 0.02536 \cdot E_{SA} - 0.1056 \cdot E_{SP} - 0.0309 \cdot E_{CA} + 0.01278 \cdot E_{CP} + 3.6124 \cdot f_Z - 01731 \yen_F$$

$$B = -1.3162 \cdot 10^{-17} \cdot E_{SA} \yen_F{}^{11.0706} + 0.1167 \cdot E_{CA} - 24.9484 \cdot E_{CP} \yen_F{}^{-1.8176}$$

$$C = 7.8172 \cdot 10^{-17} \cdot f_Z \yen_F{}^{12.6755} + 1.0392 \cdot 10^{-13} \cdot f_Z \cdot c_D{}^{22.4064} + 17.947$$

Table 6 contains calculated cutting forces and appropriate input data for the model such as: cutting depth, c_D, feed rate per tooth, f_Z, rake angle, γ_F, elasticity modulus by stretching along grains, E_{SA}, elasticity modulus by stretching perpendicular to grains, E_{SP}, modulus of elasticity by compression along grains, E_{CA}, and modulus of elasticity by compression perpendicular to grains, E_{CP}.

Table 6. *Calculated wood cutting forces – Porankiewicz model*

NO	γ_F	C_D	f_Z	Esa	Esp	Eca	Ecp	$F_{porankiewicz}$
	°	mm	mm	MPa	MPa	MPa	MPa	N
1	25	4.5	0.171	1400.68	65.81	821.1	459.7	24.38
2	16	2	0.683	1419.26	71.54	812.4	499	79.64
3	16	4.5	0.171	1385.14	61.24	838	480.6	32.93
4	20	3	0.341	1350.07	74.31	862.9	511.2	43.16
5	25	2	0.683	1375.27	68.3	788.6	519.8	46.60
6	16	2	0.171	1390.87	68.11	826.2	511.9	19.72
7	20	3	0.341	1313.09	65.53	842.5	489.5	43.37
8	25	4.5	0.683	1271.94	65.45	811.2	455.7	71.82
9	20	3	0.341	1267.57	63.44	804.5	470.2	41.16
10	20	3	0.341	1390.63	61.16	839.2	489.2	47.61
11	16	2	0.683	1297.92	54.18	782.1	492.6	56.33
12	16	4.5	0.171	1164.54	67.01	850.7	488.3	28.66
13	16	4.5	0.683	1379.08	57.19	838.9	496.3	94.46
14	20	3	0.341	1351.99	59.91	866.6	489.7	46.58
15	20	3	0.341	1350.21	64.05	804.6	484.7	42.75
16	25	4.5	0.171	1354.42	66.71	837.6	484.9	25.44
17	20	3	0.341	1312.34	66.23	812.8	466	42.73
18	25	2	0.683	1407.98	58.99	841.1	476.3	49.31
19	25	4.5	0.683	1293.22	71.91	819.5	477.8	70.32
20	16	2	0.171	1382.74	78.11	845.9	483.5	29.40

For the examined cutting conditions the minimum value of calculated cutting force was 19.720 N and the maximum value was 94.460 N according to this model.

Figure 2 shows in parallel the values of the measured and calculated values of the mean cutting forces for all measurements performed.

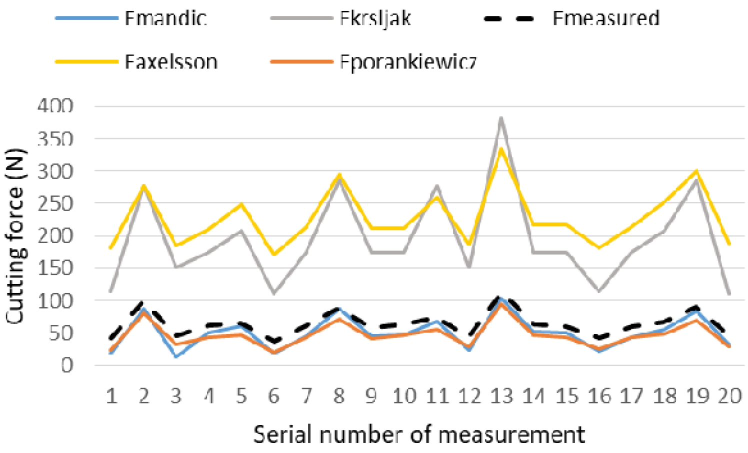

Figure 2*. Measured and calculated mean cutting forces in peripheral milling of oak*

It can be seen from the figure that there is no coincidence of the results, but that there is a similarity of the curve shape, i.e. the changes in the measured values are accompanied by corresponding changes in the calculated values. Significantly better situation is reflected in Mandić and Porankiewicz models where comparative analysis of calculated and measured values established not only similar behavior, i.e. similar response to change in the cutting parameters, but also a better match of the measured and calculated cutting force values.

Comparing the results from tables 3, 4, 5 and 6, it can be seen that the differences between the forces calculated by these models are relatively large and differ substantially from those measured, which will be commented below. Statistical comparison between measured and calculated values is performed based on two parameters: the ratio between mean values of the sets of measured and calculated values (systemic difference between measured and calculated values) and the ratio between variances of the sets of measured and calculated values (Table 7).

Table 7. *Statistical analysis of data*

Force (N)	**Valid N**	**Mean**	**Minimum**	**Maximum**	**Variance**	**Std.Dev.**	**Coef.Var.**
Fmandić	20	50.396	13.070	103.930	661.281	25.715	51.027
Fkršljak	20	195.181	110.426	382.264	5254.622	72.489	37.139
Fmeasured	20	64.180	36.288	113.288	414.944	20.370	31.739
Faxelsson	20	227.660	171.219	333.490	2084.192	45.653	20.053
Fporankiewicz	20	46.819	19.724	94.462	380.741	19.513	41.677

Comparison between results presented in tables 3 and 4 shows that differences between forces calculated using Kršljak and Axelsson models are relatively high and deviate considerably from the measured ones. Figure 3 displays in parallel data on mean cutting force per a single blade and force calculated using the method of coefficients (Kršljak's model), i.e. the Axelsson's model.

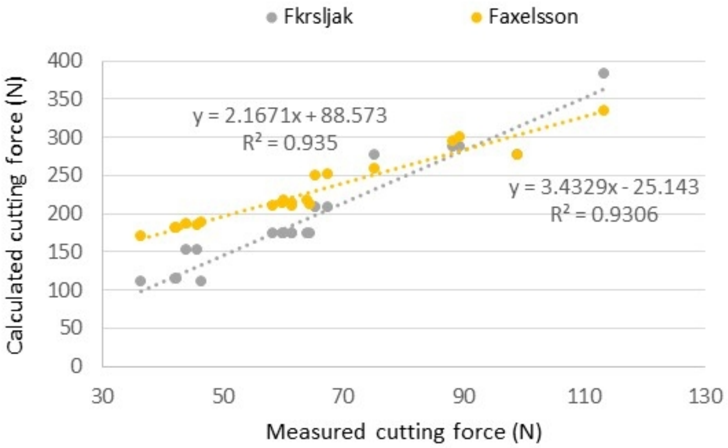

Figure 3. *The ratio between measured and calculated values of cutting force for both models.*

The linear regression equation for the method of coefficients has the following form:

$$F_{krsljak} = 3.4329 \cdot F_{measured} - 25.143 \quad (8)$$

Based on the given equation and obtained coefficient of determination (R^2 = 0.930), it can be concluded that there is a strong correlation between measured and calculated values, but they do not coincide. The coefficient of direction in the regression equation equals 3.433, the ratio between mean measured and calculated values equals 3.041 and the ratio of variations equals 3.55 points to a significant difference between measured and calculated values (Table 6). Similar situation can be seen in the linear regression equation for Axelsson method:

$$F_{axelsson} = 2.1671 \cdot F_{measured} + 88.573 \tag{9}$$

Axelsson's model proved to be somewhat better for prediction of cutting forces in peripheral milling. The coefficient of determination ($R^2 = 0.935$) is very high in this case too and indicates a very strong correlation between measured and calculated values. However, the coefficient of direction in the regression equation equals 2.167, the ratio between mean measured and calculated values equals 3.547and the ratio of variations equals 2.241, which indicates that the difference between measured and calculated forces remains high (Table 6). It can be concluded that the presented models are suitable for application if the comparison is done between the impacts of some factors on the mechanics of cutting, but they are not suitable for quantification of cutting force values. Comparison between results presented in tables 5 and 6 shows that differences between forces calculated using these two models are relatively low, but they still exist. The equations presented in Figure 4 provide a strong link between the observed cutting force and the predicted cutting force and can certainly be used to analyze the influence of specific inputs on the predicted cutting forces according to very high coefficients of determination.

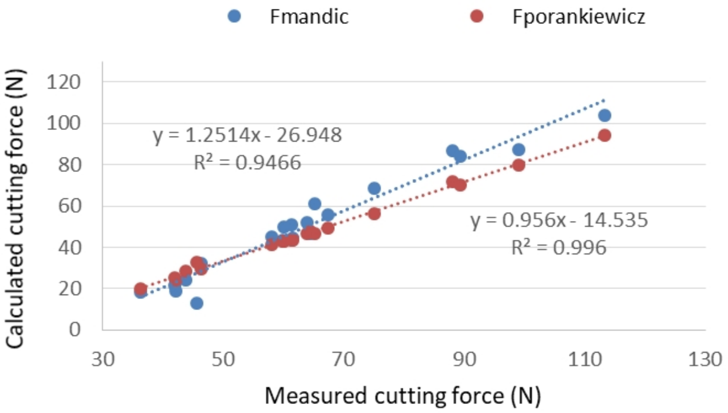

Figure 4. *The ratio between measured and calculated values of cutting force for both models*

The linear regression equation for the Mandić prediction model has the form:

$$F_{mandic} = 1.251 \cdot F_{measured} - 26.948 \tag{10}$$

Based on the given equation and obtained coefficient of determination ($R^2 = 0.947$), it can be concluded that there is a strong correlation between measured and calculated values. The coefficient of direction in the regression equation equals 1.251 and the ratio between mean measured and calculated values equals 0.785 the ratio of variations equals 1.26 which points to a significant difference between measured and calculated values.

Similar situation can be seen in linear regression equation for the Porankiewicz method:

$$F_{porankiewicz} = 0.956 \cdot F_{measured} - 14.535 \tag{11}$$

The Porankiewicz's model proved to be somewhat better for prediction of cutting forces in peripheral milling. The coefficient of determination ($R^2 = 0.996$) is very high in this case too, and indicates a very strong correlation between measured and calculated values. However, the coefficient of direction in the regression equation equals 0.956, the ratio between mean measured and calculated values equals1.371 and the ratio of variations equals 0.957 which indicates that difference between measured and calculated forces still remains. It can be concluded that these equations (for Mandić and Porankiewicz models) provide a strong link between the measured and predicted cutting forces and can be used to analyze the impact of specific inputs on the predicted cutting forces. For example, in Mandić model an almost linear relationship between predicted main force and density and predicted main force and moisture content is shown (Figure 5).

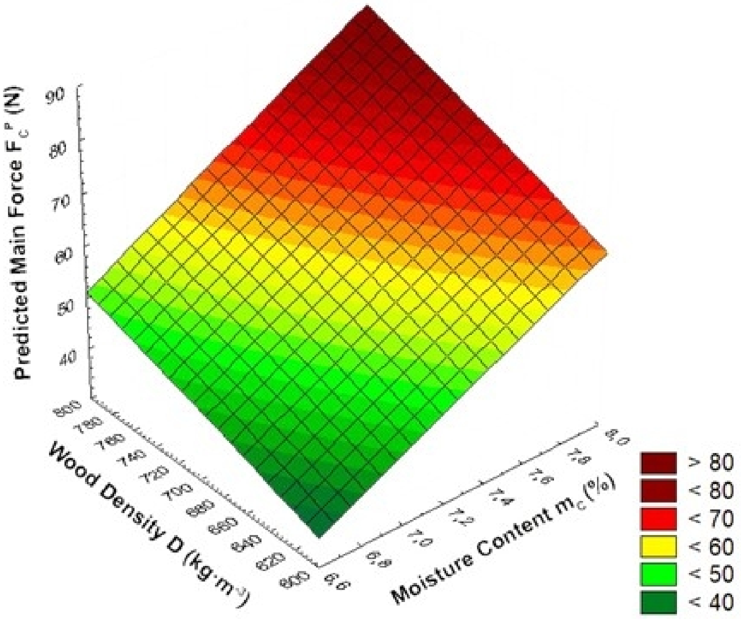

Figure 5. *Plot of the relationships between the predicted main force, F_C^P and D and m_C, according to Mandić model (Mandić, Porankiewicz, Danon 2015)*

The predicted main cutting force F_C^P depends on the increase in density D and m_C. If moisture content increase is 0.1 %, the average increase in force will be around 2 N. If density increase is 10 kg·m^{-3}, the average increase in force, will be around 0.8 N (Mandić, Porankiewicz, Danon 2015).

Based on the given equation in Mandić's model (Equation 6), the dependences of wood density and moisture content in relation to the cutting forces can be found. The equation presented in Figure 6 provides a strong link between the calculated cutting force and the wood density, and it can certainly be used to analyze the influence of this input on the cutting forces according to high coefficients of determination ($R^2 = 0.8985$).

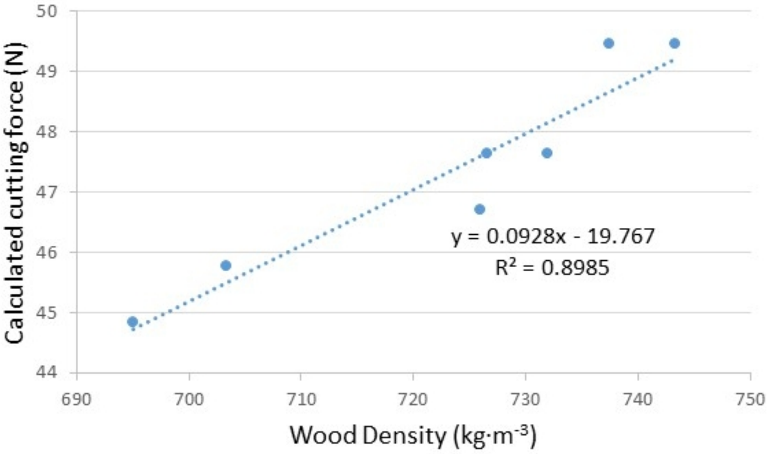

Figure 6. *The ratio between wood density and calculated cutting force according to Mandić model*

The linear regression equation has the form:

$$F_{mandic} = 0.0928 \cdot D - 19.767 \tag{12}$$

If density increase is 10 kg·m^{-3}, the average increase in force will be around 0.928 N. This equation can be used to analyze the influence of wood density on the cutting forces under real cutting conditions in the reference range. The equation presented in figure 7 provides a strong link between the calculated cutting force and the moisture content according to high coefficients of determination ($R^2 = 0.8175$).

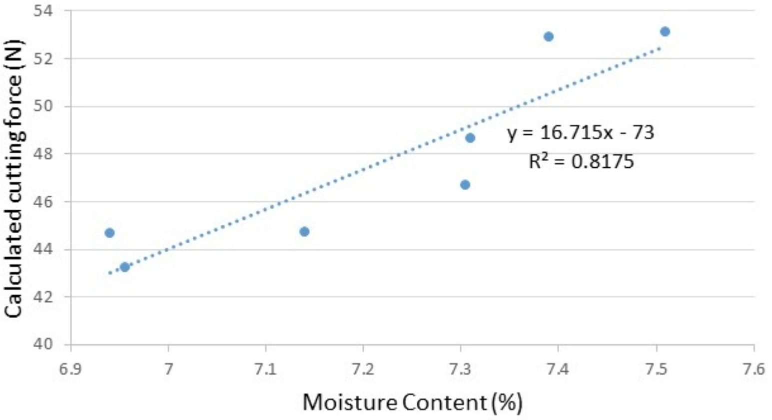

Figure 7. *The ratio between moisture content and calculated cutting force according to Mandić model*

The linear regression equation has the form:

$$F_{mandic} = 16.715 \cdot m_C - 73 \qquad (13)$$

If moisture content increase is 0.1 %, the average increase in force will be around 1.67 N. This equation can be used to analyze the influence of moisture content on the cutting forces in the actual cutting conditions in the reference range.

4. CONCLUSIONS

Comparative analysis of calculated and measured values established similar behavior, i.e. similar response to change in the cutting parameters, primarily mean chip thickness and mean angle between cutting direction and wood grain orientation.

Between measured values of mean force in peripheral milling and those calculated by the method of coefficients for the same input data (Kršljak, 2013) there is a very strong correlation (coefficient of determination is 0.93), but there is no coincidence. The coefficient of direction in the regression equation amounts to 3.433, whereas calculated values are, on average, by 3.041 times as high as those measured.

The Axelsson model proved a very high coefficient of determination ($R^2 = 0.935$) which points to a very strong correlation between measured and calculated values. However, the difference between measured and calculated values remains high. The coefficient of direction in the regression equation amounts to 2.167, whereas calculated values are, on average, by 3.547 times as high as those measured.

The Mandić and Porankiewicz models proved to be somewhat better for prediction of cutting forces in peripheral milling. Between measured values of mean force in peripheral milling and those calculated by the Mandić model for the same input data there is a very strong correlation ($R^2 = 0.947$) but there is no coincidence. Calculated values are, on average, by 1.2 times as low as those measured. Porankiewicz model showed a very high coefficient of determination ($R^2 = 0.996$), which points to a very strong correlation between measured and calculated values, but the fact that calculated values are, on average, by 1.371 times as low as those measured indicates that the difference between measured and calculated forces still remains.

Finally, it can be concluded, despite the fact that given models are not suitable for quantification of specific values of the cutting forces, that they are a simple tool suitable for application to analyze the impact of specific inputs on the predicted cutting forces.

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